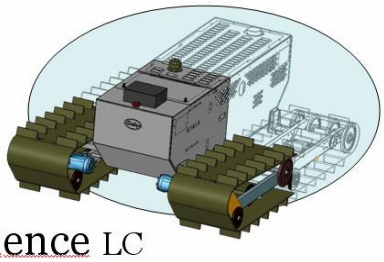
Form: i02135
Rev: 01
Date: 20 Sept 2011

*Company Proprietary*

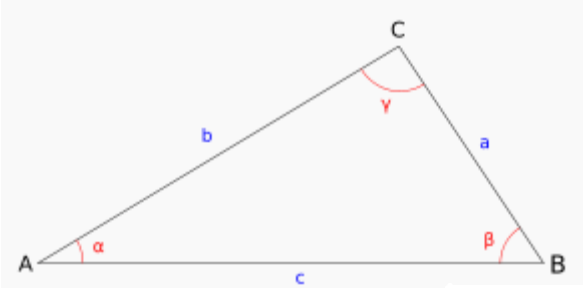


Invenscience LC

*The science of invention*

Equations of Motion for ARM 2.0

The intent is to derive the angular velocity of DOF 2, 3 and 4 as a function of the linear position of the servos and as a function of the angular position of each segment.

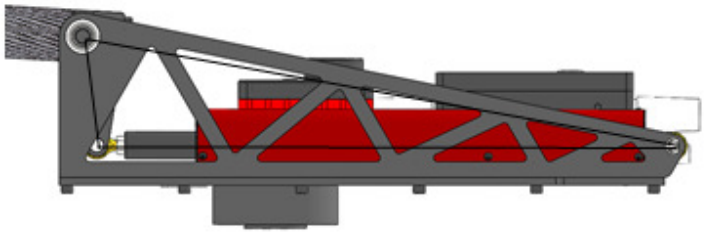



Law of cosines in its generic form: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

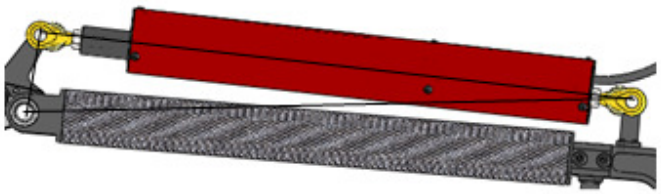
For the three cases below, c will represent the length of the actuator.

DOF 3

DOF 2



DOF 4



$$c(t)^2 = a^2 + b^2 - 2\cdot a\cdot c(t)\cdot\cos(\gamma)\ \text{solve},\gamma \rightarrow \begin{bmatrix} \operatorname{acos}\left(\frac{a^2 + b^2 - c(t)^2}{2\cdot a\cdot c(t)}\right) \\ -\operatorname{acos}\left[\frac{\frac{1}{2}}{\frac{a}{c(t)}}\cdot\left(a^2 + b^2 - c(t)^2\right)\right] \end{bmatrix}$$

$$\frac{d}{dt}\operatorname{acos}\left(\frac{a^2 + b^2 - c(t)^2}{2\cdot a\cdot c(t)}\right) \rightarrow \frac{\frac{\frac{d}{dt}c(t)}{a} + \frac{\frac{d}{dt}c(t)\cdot\left(a^2 + b^2 - c(t)^2\right)}{2\cdot a\cdot c(t)^2}}{\sqrt{1 - \frac{\left(a^2 + b^2 - c(t)^2\right)^2}{4\cdot a^2\cdot c(t)^2}}}$$

Find the Angular Velocity as a funtion of the linear position of the servo

$c\_dot\_dof2 := .3\,\frac{in}{s} = 7.62\times 10^{-3}\,\frac{m}{s}$ $\qquad$ $c\_dof2\_min := 17.29\,in$

$a\_dof2 := 3.5\,in = 0.089\,m$ $\qquad$ $c\_dof2\_max := 23.29\,in$

$b\_dof2 := 19.2\,in = 0.488\,m$

$$\gamma\_dot\_dof2(c\_dof2) := \frac{\frac{c\_dot\_dof2}{a\_dof2} + \frac{c\_dot\_dof2\cdot\left(a\_dof2^2 + b\_dof2^2 - c\_dof2^2\right)}{2\cdot a\_dof2\cdot c\_dof2^2}}{\sqrt{1 - \frac{\left(a\_dof2^2 + b\_dof2^2 - c\_dof2^2\right)^2}{4\cdot a\_dof2^2\cdot c\_dof2^2}}}$$

$c\_dot\_dof3 := .6\,\frac{in}{s} = 0.015\,\frac{m}{s}$ $\qquad$ $c\_dof3\_min := 17.8\,in$

$a\_dof3 := 3.14\,in = 0.08\,m$ $\qquad$ $c\_dof3\_max := 23.29\,in$

$b\_dof3 := 20.35\,in = 0.517\,m$

$$\gamma\_dot\_dof3(c\_dof3) := \frac{\frac{c\_dot\_dof3}{a\_dof3} + \frac{c\_dot\_dof3\cdot\left(a\_dof3^2 + b\_dof3^2 - c\_dof3^2\right)}{2\cdot a\_dof3\cdot c\_dof3^2}}{\sqrt{1 - \frac{\left(a\_dof3^2 + b\_dof3^2 - c\_dof3^2\right)^2}{4\cdot a\_dof3^2\cdot c\_dof3^2}}}$$

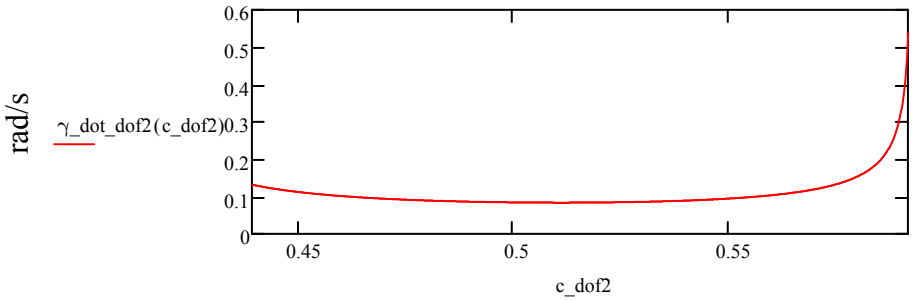
$c\_dot\_dof4 := .6\,\frac{in}{s} = 0.015\,\frac{m}{s}$ $\qquad$ $c\_dof4\_min := 17.29\,in$

$a\_dof4 := 2.45\,in = 0.062\,m$ $\qquad$ $c\_dof4\_max := 20.29\,in$

$b\_dof4 := 18.77\,in = 0.477\,m$

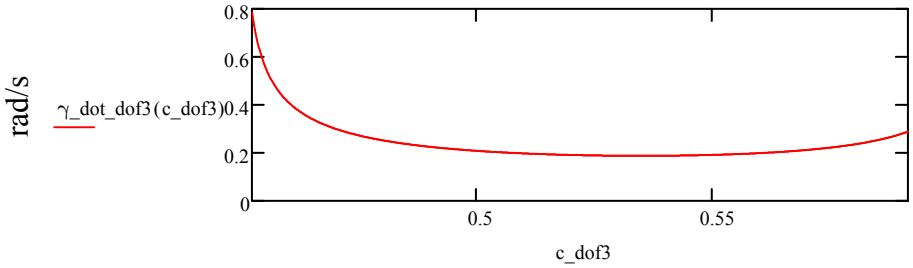
$$\gamma\_dot\_dof4(c\_dof4) := \frac{\frac{c\_dot\_dof4}{a\_dof4} + \frac{c\_dot\_dof4\cdot\left(a\_dof4^2 + b\_dof4^2 - c\_dof4^2\right)}{2\cdot a\_dof4\cdot c\_dof4^2}}{\sqrt{1 - \frac{\left(a\_dof4^2 + b\_dof4^2 - c\_dof4^2\right)^2}{4\cdot a\_dof4^2\cdot c\_dof4^2}}}$$

Angular Velocity of DOF 2



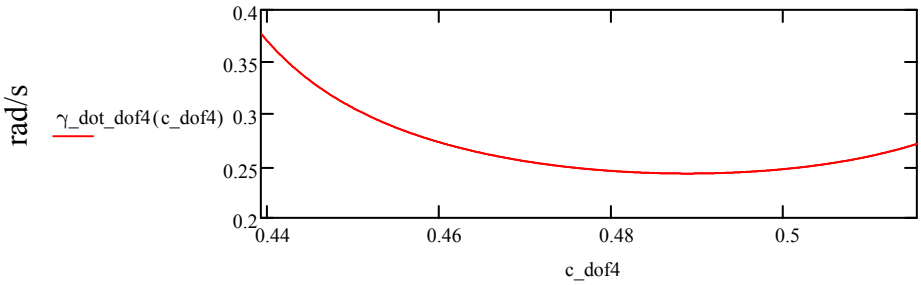

Servo linear position (in)

Angular Velocity of DOF 3




Servo linear position (in)

Angular Velocity of DOF 4




Servo linear position (in)

Find the Angular Velocity as a funtion of the angular positions.

$$\gamma\_dof2\_min := \operatorname{acos}\left(\frac{a\_dof2^2 + b\_dof2^2 - c\_dof2\_min^2}{2 \cdot a\_dof2 \cdot c\_dof2\_min}\right) = 0.827$$

$$\gamma\_dof2\_max := \operatorname{acos}\left(\frac{a\_dof2^2 + b\_dof2^2 - c\_dof2\_max^2}{2 \cdot a\_dof2 \cdot c\_dof2\_max}\right) = 3.006$$

$$c\_dof2(\gamma\_dof2) := \frac{\sqrt{2} \cdot \sqrt{3 \cdot a\_dof2^2 + 2 \cdot b\_dof2^2 + a\_dof2^2 \cdot \cos(2 \cdot \gamma\_dof2)}}{2} - a\_dof2 \cdot \cos(\gamma\_dof2)$$

$$\gamma\_dot\_dof2\_alt(\gamma\_dof2) := \frac{\frac{c\_dot\_dof2}{a\_dof2} + \frac{c\_dot\_dof2 \cdot \left(a\_dof2^2 + b\_dof2^2 - c\_dof2(\gamma\_dof2)^2\right)}{2 \cdot a\_dof2 \cdot c\_dof2(\gamma\_dof2)^2}}{\sqrt{1 - \frac{\left(a\_dof2^2 + b\_dof2^2 - c\_dof2(\gamma\_dof2)^2\right)^2}{4 \cdot a\_dof2^2 \cdot c\_dof2(\gamma\_dof2)^2}}}$$

$$\gamma\_dof3\_min := \operatorname{acos}\left(\frac{a\_dof3^2 + b\_dof3^2 - c\_dof3\_min^2}{2 \cdot a\_dof3 \cdot c\_dof3\_min}\right) = 0.289$$

$$\gamma\_dof3\_max := \operatorname{acos}\left(\frac{a\_dof3^2 + b\_dof3^2 - c\_dof3\_max^2}{2 \cdot a\_dof3 \cdot c\_dof3\_max}\right) = 2.515$$

$$c\_dof3(\gamma\_dof3) := \frac{\sqrt{2} \cdot \sqrt{3 \cdot a\_dof3^2 + 2 \cdot b\_dof3^2 + a\_dof3^2 \cdot \cos(2 \cdot \gamma\_dof3)}}{2} - a\_dof3 \cdot \cos(\gamma\_dof3)$$

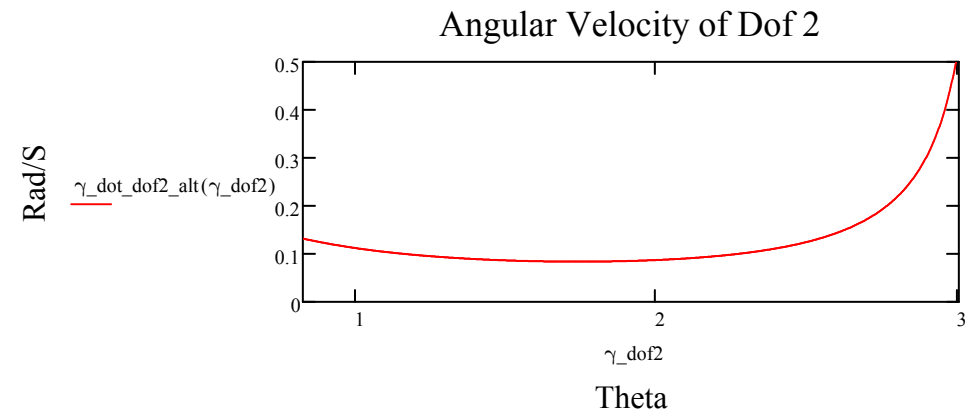
$$\gamma\_dot\_dof3\_alt(\gamma\_dof3) := \frac{\frac{c\_dot\_dof3}{a\_dof3} + \frac{c\_dot\_dof3 \cdot \left(a\_dof3^2 + b\_dof3^2 - c\_dof3(\gamma\_dof3)^2\right)}{2 \cdot a\_dof3 \cdot c\_dof3(\gamma\_dof3)^2}}{\sqrt{1 - \frac{\left(a\_dof3^2 + b\_dof3^2 - c\_dof3(\gamma\_dof3)^2\right)^2}{4 \cdot a\_dof3^2 \cdot c\_dof3(\gamma\_dof3)^2}}}$$

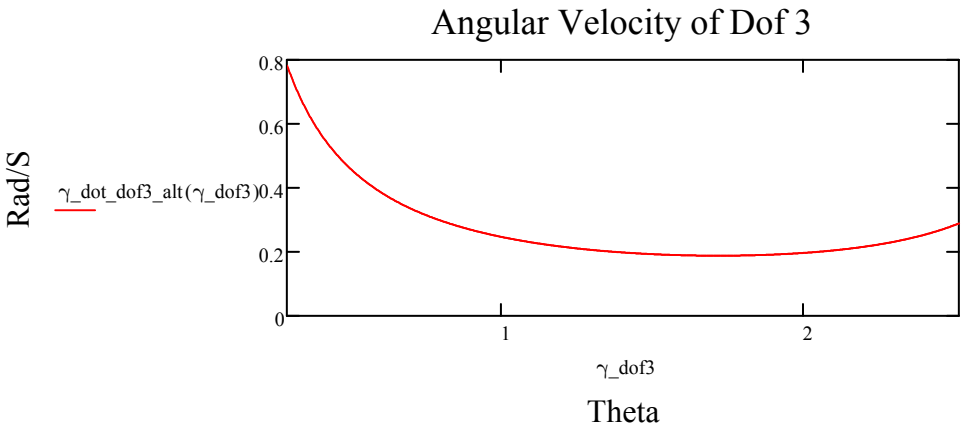
$$\gamma\_dof4\_min := \operatorname{acos}\left(\frac{a\_dof4^2 + b\_dof4^2 - c\_dof4\_min^2}{2 \cdot a\_dof4 \cdot c\_dof4\_min}\right) = 0.794$$

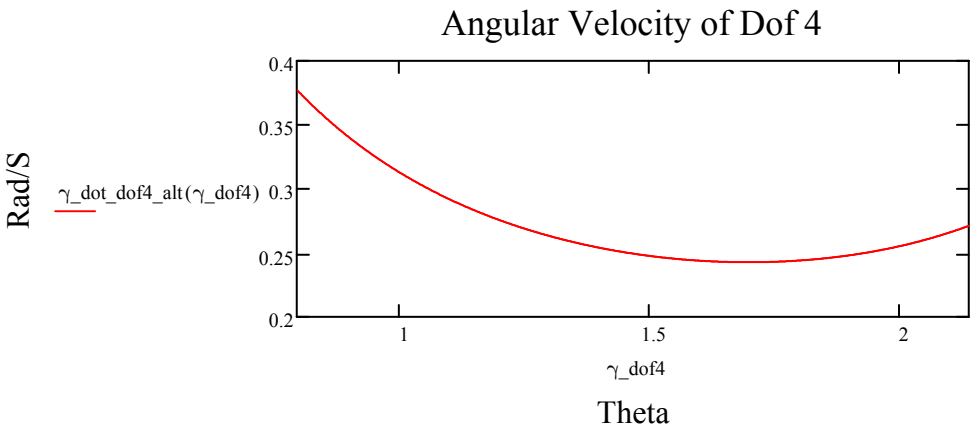
$$\gamma\_dof4\_max := \operatorname{acos}\left(\frac{a\_dof4^2 + b\_dof4^2 - c\_dof4\_max^2}{2 \cdot a\_dof4 \cdot c\_dof4\_max}\right) = 2.137$$

$$c\_dof4(\gamma\_dof4) := \frac{\sqrt{2} \cdot \sqrt{3 \cdot a\_dof4^2 + 2 \cdot b\_dof4^2 + a\_dof4^2 \cdot \cos(2 \cdot \gamma\_dof4)}}{2} - a\_dof4 \cdot \cos(\gamma\_dof4)$$

$$\gamma\_dot\_dof4\_alt(\gamma\_dof4) := \frac{\frac{c\_dot\_dof4}{a\_dof4} + \frac{c\_dot\_dof4 \cdot \left(a\_dof4^2 + b\_dof4^2 - c\_dof4(\gamma\_dof4)^2\right)}{2 \cdot a\_dof4 \cdot c\_dof4(\gamma\_dof4)^2}}{\sqrt{1 - \frac{\left(a\_dof4^2 + b\_dof4^2 - c\_dof4(\gamma\_dof4)^2\right)^2}{4 \cdot a\_dof4^2 \cdot c\_dof4(\gamma\_dof4)^2}}}$$


Angular Velocity of Dof 2
Rad/S
γ_dot_dof2_alt(γ_dof2)
0.5
0.4
0.3
0.2
0.1
0
1
2
3
γ_dof2
Theta


Angular Velocity of Dof 3
Rad/S
γ_dot_dof3_alt(γ_dof3)
0.8
0.6
0.4
0.2
0
1
2
γ_dof3
Theta


Angular Velocity of Dof 4
Rad/S
γ_dot_dof4_alt(γ_dof4)
0.4
0.35
0.3
0.25
0.2
1
1.5
2
γ_dof4
Theta